

# Cornell Notes

## Main Ideas/Questions

Name: \_\_\_\_\_  
Class/Period: \_\_\_\_\_  
Date: \_\_\_\_\_

Topic/ Objective: \_\_\_\_\_

## 5.3 Concurrent Lines, Medians, and Altitudes

### Theorem 5-6

The perpendicular bisectors of the sides of a triangle are

Concurrent at a point equidistant from the vertices

### Theorem 5-7

The bisectors of the angles of a triangle are

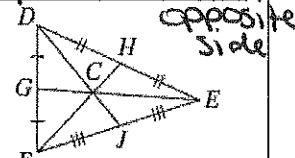
Concurrent at a point equidistant from the sides

### Theorem 5-8

The medians of a triangle are concurrent at a point that is two thirds the distance from each vertex to the midpoint of the opposite side

$$DC = \frac{2}{3} DJ \quad EC = \frac{2}{3} EG \quad FC = \frac{2}{3} FH$$

↑ median      ↑ median      ↑ median



### Theorem 5-9

The lines that contain the altitudes of a triangle are concurrent

Definitions:

Concurrent: 3 or more lines intersect in one point

Point of concurrency: The point at which the lines intersect

Circumcenter of the triangle: The point of concurrency of the  $\perp$  bisectors of a triangle (C)

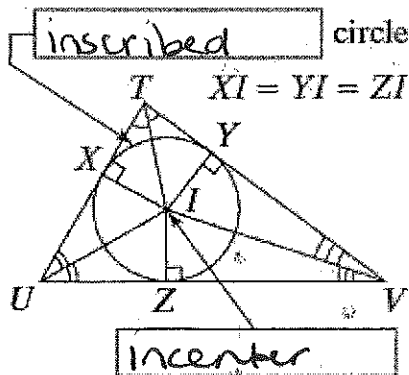
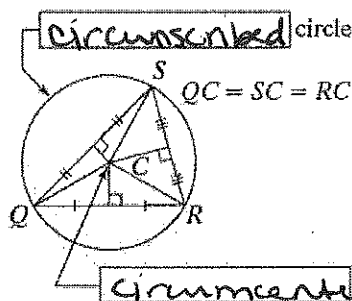
Circumscribed: All points of the  $\Delta$  touch the circle and are equidistant from the point of concurrency

Inscribed: points X, Y, Z are equidistant from I (circle inside figure)

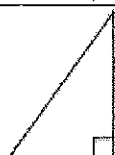
Incenter of the triangle: point of concurrency of the angle bisectors of a  $\Delta$  (I)

Altitude:  $\perp$  segment from a vertex to the line containing the opposite side

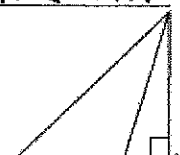
Orthocenter: The lines containing the altitudes of a  $\Delta$  are concurrent at this point



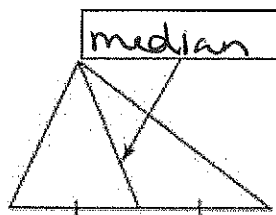
Acute Triangle  
Altitude is  
Inside



Right Triangle  
Altitude is  
A side



Obtuse Triangle  
Altitude is  
outside



Median: Segment whose endpoints are a vertex and the midpoint of the opposite side

Centroid: The point of concurrency of the medians of a  $\Delta$

### Example 1

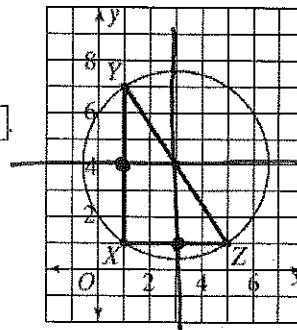
**Finding the Circumcenter** Find the center of the circle that circumscribes  $\Delta XYZ$ .

Because  $X$  has coordinates  $(4, 1)$  and  $Y$  has coordinates  $(1, 7)$ ,  $\overline{XY}$  lies on the vertical line  $x = 1$ .

The perpendicular bisector of  $\overline{XY}$  is the horizontal line that passes through  $(1, \frac{1+7}{2})$  or  $(1, 4)$ , so the equation of the perpendicular bisector of  $\overline{XY}$  is  $y = 4$ .

Because  $X$  has coordinates  $(4, 1)$  and  $Z$  has coordinates  $(5, 1)$ ,  $\overline{XZ}$  lies on the horizontal line  $y = 1$ . The perpendicular bisector of  $\overline{XZ}$  is the vertical line that passes through  $(\frac{4+5}{2}, 1)$  or  $(3, 1)$ , so the equation of the perpendicular bisector of  $\overline{XZ}$  is  $x = 3$ .

Draw the lines  $y = 4$  and  $x = 3$ . They intersect at the point  $(3, 4)$ . This point is the center of the circle that circumscribes  $\Delta XYZ$ .



### Example 2

**Finding Lengths of Medians**  $M$  is the centroid of  $\Delta WOR$ , and  $WM = 16$ . Find  $WX$ .

The Centroid  $M$  is the point of concurrency of the medians of a triangle.

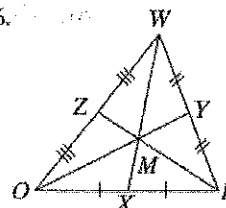
The medians of a triangle are concurrent at a point that is  $\frac{2}{3}$  the distance from each vertex to the midpoint of the opposite side. (Theorem 5-8)

Because  $M$  is the Centroid of  $\Delta WOR$ ,  $WM = 16$ .

$$WM = \frac{2}{3} WX \quad \text{Theorem 5-8}$$

$$16 = \frac{2}{3} WX \quad \text{Substitute } 16 \text{ for } WM.$$

$$24 = WX \quad \text{Multiply each side by } \frac{3}{2}$$



### Example 3

Find the center of the circle that you can circumscribe about the triangle with vertices  $(0, 0)$ ,  $(-8, 0)$ , and  $(0, 6)$ .

$$(-4, 3)$$

