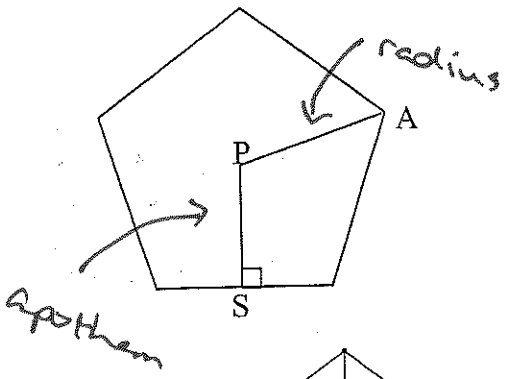
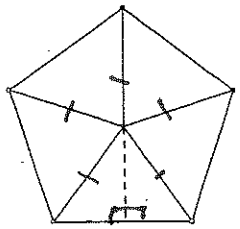
Notes for sections 10.3 and 10.5 – Area of Regular Polygons

In a regular polygon, a segment drawn from the center of the polygon perpendicular to a side of the polygon is called an apothem. In the figure at the right $\overline{PS}$ is an apothem. A segment drawn from the center of the polygon to a vertex is called a radius of the polygon. In the figure at the right, $\overline{PA}$ is a radius.



The area of a regular polygon can be found by dividing the polygon into congruent isosceles triangles. For example, the pentagon above can be divided into 5 triangles by drawing all five radii. If a regular polygon has n sides then it can be divided into n triangles.



Now find the area of one of the triangles (note: area of triangle = ½ bh). The height of the triangle will be the apothem. The base of the triangle is the length of one side, s, of the polygon. Therefore, area of the triangle = ½ sa.

Since there are n triangles, multiply the area of the triangle by n to get the area of the polygon.

Area of polygon = n[1/2s·a] or ½ n·s·a

However, n·s is just the perimeter, P, of the polygon. Therefore, the area of a regular polygon with perimeter P and apothem a is

A = ½ Pa

Ex. 1 Find the area of a regular pentagon with perimeter 54.49 m and an apothem of 7.5 m.

$A = \frac{1}{2}(7.5)(54.49)$

$= 204.3375\ m^2$

Ex. 2 Find the area of a regular hexagon with an apothem of $5\sqrt{3}$ cm and each side 10 cm.

$A = \frac{1}{2}(5\sqrt{3})(60)$

$= 150\sqrt{3}\ cm^2$

Equilateral triangles are sometimes easier to work with $A = \frac{s^2\sqrt{3}}{4}$

Central angle: $\frac{360}{n}$

Ex. 3 Find the apothem, area, and perimeter of the equilateral triangle below.

$a = 7$

$P = 42\sqrt{3}$

$A = \frac{1}{2}(7)(42\sqrt{3})$

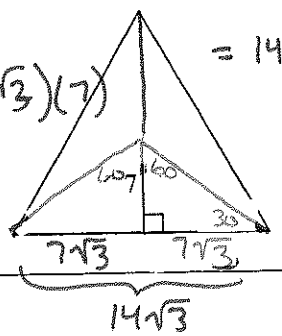
$= 147\sqrt{3}$

Equilateral △

$a = 7$

$A = \frac{1}{2}(42\sqrt{3})(7)$

$P = 42\sqrt{3}$

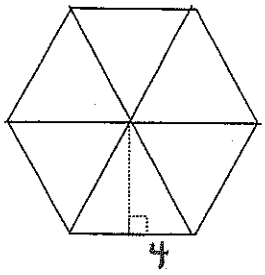


$= 147\sqrt{3}$ units²

Ex. 4 Hexagons are made up of 6 congruent equilateral triangles so the area of a regular hexagon can be found using: $A = 6 \times \left(\frac{s^2\sqrt{3}}{4} \right)$. Find the apothem, perimeter, and area of a regular hexagon that has a side of 8 in.

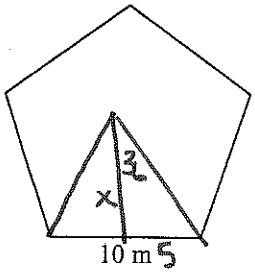
$A = 6\left(\frac{64\sqrt{3}}{4}\right)$

$= 96\sqrt{3}\ in^2$



Find the apothem, perimeter, and area of each regular polygon. Round your answers to the nearest tenth.

Ex. 5



2nd Ans

$\tan 36 = \frac{5}{x}$

$x = 6.9$

$P = 50$

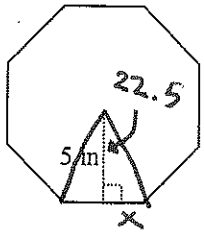
$A = \frac{1}{2}(6.9)(50)$

$= 172\ m^2$

$a = 6.9$

$P = 50$

Ex. 6



$\tan 22.5 = \frac{x}{5}$

$x = 2.1$

$P = 33.1$

$A = \frac{1}{2}(5)(33.1)$

$= 82.8\ in^2$