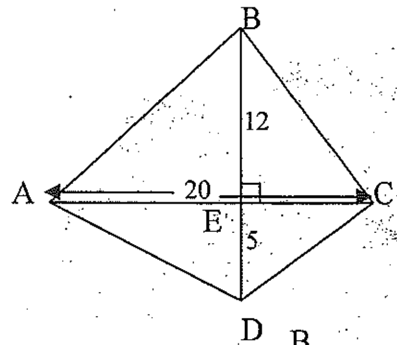
Notes for 10-2

Area of Quadrilateral ABCD with Perpendicular Diagonals



Area of Quadrilateral ABCD = Area of ΔABC + Area of ΔADC

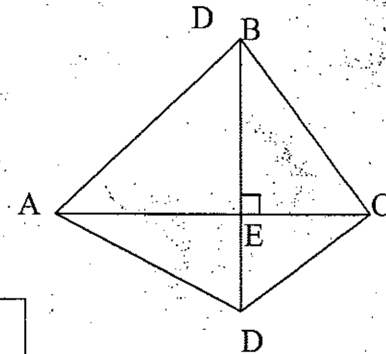
Area of Quadrilateral ABCD = $\frac{1}{2}(20)(12) + \frac{1}{2}(20)(5) =$

$120 + 50 = 170$ square units

or

Area of Quadrilateral ABCD = $\frac{1}{2}(20)(12 + 5) =$

$\frac{1}{2}(20)(17) = 170$ square units



Area of Quadrilateral ABCD = $\frac{1}{2}(AC)(BD)$

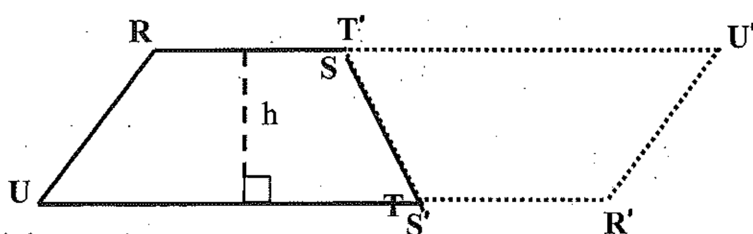
Area of Quadrilateral ABCD = $\frac{1}{2}$(the product of the diagonals)

Area of a Quadrilateral with Perpendicular Diagonals = $\frac{1}{2}d_1d_2$

What special quadrilaterals have perpendicular diagonals?

$Area_{rhombus} = \frac{1}{2}d_1d_2$	$Area_{kite} = \frac{1}{2}d_1d_2$	$Area_{square} = \frac{1}{2}d^2$ (Diagonals of a square are $\cong$)

Area of Trapezoid RSTU

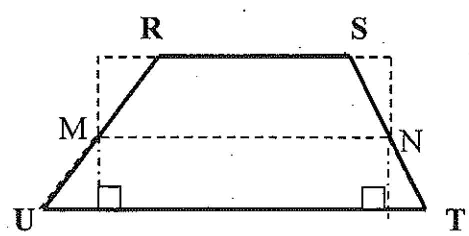


Copy trapezoid RSTU and call it R'S'T'U'. Rotate the copy around the midpoint of segment ST. The resulting figure is a parallelogram RU'R'U. The original trapezoid RSTU is half of that parallelogram.

$A_{trapezoid} = \frac{1}{2}(h)(RS + TU)$

$A_{trapezoid} = \frac{1}{2}(h)(b_1 + b_2)$ where h is height and b_1 and b_2 are the bases

or

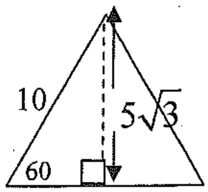


Construct $\overline{MN}$, the median of trapezoid RSTU.
Drop perpendiculars from M and N to base $\overline{UT}$.
Rotate the small triangles that are formed around the midpoints, M and N. A rectangle with length MN is formed.

$A_{trapezoid}$ = (median)(height)

Area of Equilateral Triangle

Equilateral triangle with sides of 10 cm. each -

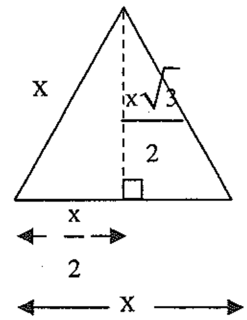


$$A_{triangle} = \frac{1}{2}bh$$

$$= \frac{1}{2}(10)(5\sqrt{3})$$

$$= 25\sqrt{3}$$

In general -



$$A_{triangle} = \frac{1}{2}bh$$

$$= \frac{1}{2}(x)\left(\frac{x\sqrt{3}}{2}\right)$$

$$= \frac{x^2\sqrt{3}}{4}$$

$$\boxed{A_{equilateral\ \Delta} = \frac{x^2\sqrt{3}}{4} \text{ where } x \text{ is side of } \Delta}$$

Example 1

Find the height of a trapezoid that has an area of 287 square inches and bases of 38 inches and 44 inches.

$$A_{trapezoid} = \frac{1}{2}h(b_1 + b_2)$$

$$287 = \frac{1}{2}h(38 + 44)$$

$$574 = h(82)$$

$$574/82 = h$$

$$7 = h$$

Example 2

Find the height of a trapezoid that has an area of 84 cm^2 if its median is 12 cm.

$$A_{trapezoid} = (\text{median})(\text{height})$$

$$84 = 12h$$

$$84/12 = h$$

$$7 \text{ cm.} = h$$

Example 3

Sonja wants to place a decorative brick edging around a flower garden that is in the shape of a rhombus. One diagonal is 30 feet long, and the area is 600 square feet. How many bricks must she purchase if each brick is one foot long?

$$Area_{rhombus} = \frac{1}{2}d_1d_2$$

$$600 = \frac{1}{2}(30)\, d_2$$

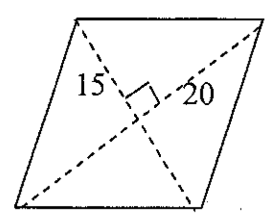
$$600 = 15\, d_2$$

$$40 = d_2$$

$$15^2 + 20^2 = side^2$$

$$25 = side$$

$$4(25) = 100 \text{ ft or } 100 \text{ bricks}$$



Example 4

Find the area of an equilateral triangle with perimeter 60 cm.

Side is $60/3 = 20$

Area is $\frac{20^2\sqrt{3}}{4} = \frac{400\sqrt{3}}{4} = 100\sqrt{3}\ cm^2$.