Geometry
Coordinates in Space

Name KEY
Date _____ Period _____

1. On which axis does each of the following points lie?

	x axis	y axis	z axis
a) (5,0,0)	✓		
b) (0,0,-2)			✓
c) (0,3,0)		✓	
d) (0,0,0)	✓	✓	✓

2. On which plane does each of the following points lie?

	XY plane	YZ plane	XZ plane
a) (0,4,6)		✓	
b) (-2,-1,0)	✓		
c) (3,0,-5)			✓
d) (0,-2,3)		✓	

3. Write an equation for each of the following planes:

a) XY plane
$z = 0$

b) YZ plane
$x = 0$

c) XZ plane
$y = 0$

Match each of the following to a description of its graph. (include all descriptions that apply)

4. (3,0,0) ADFP
5. (0,0,0) ABCDEFP
6. (0,-6,0) BDEP
7. (0,0,20) CEFP
8. (2,3,-1) P
9. (2,4,0) PD
10. (-1,0,-1) FP
11. (0,0,4) CEFP
12. x = -2 KRSGI
13. y = 6 L,Q,S,G,H
14. z = 0 M,H,I
15. z = 7 J,H,I,Q,R
16. x = 0 N,G,I
17. 2x+3y=6 G,S
18. 4x-2y=8 G,S
19. 2x+5z=10 I,R
20. 7y-2z=14 H,Q

A. point on the X-axis (x,0,0)
B. point on the Y-axis (0,y,0)
C. point on the Z-axis (0,0,z)
D. point on the XY plane (x,y,0)
E. point on the YZ plane (0,y,z)
F. point on the XZ plane (x,0,z)
G. plane ⊥ to XY plane
H. plane ⊥ to YZ plane
I. plane ⊥ to XZ plane

Choose with same letters (2)

J. plane ∥ to XY plane
K. plane ∥ to YZ plane
L. plane ∥ to XZ plane

Choose without letter not equal 0

M. XY plane
N. YZ plane
O. XZ plane

Choose without letter =0

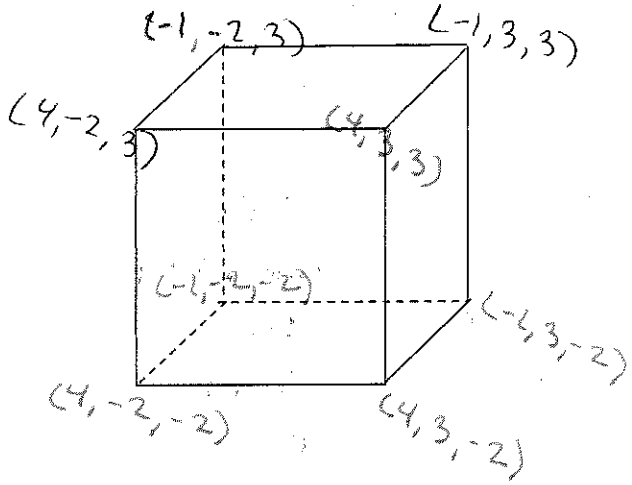
P. Point in space
Q. Plane ∥ to x-axis
R. Plane ∥ to y-axis
S. Plane ∥ to z-axis

Choose w/o letter

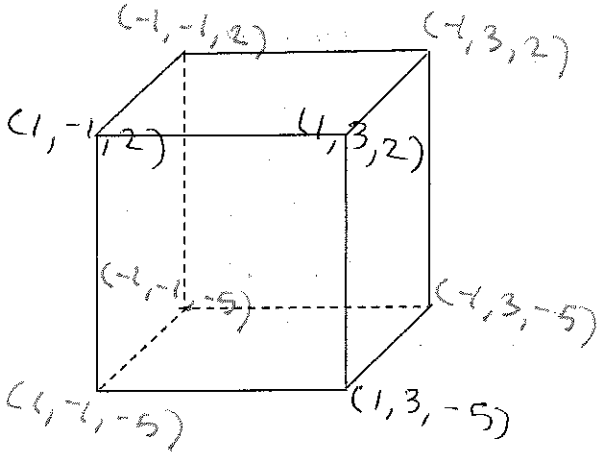
21. Name three points on the graph of each:

(a) $3x - 2y + 4z = 12$	(4, 0, 0)	(0, −6, 0)	(0, 0, 3)
(b) $7x + 4y - 14z = 28$	(4, 0, 0)	(0, 7, 0)	(0, 0, −2)
(c) $3x - 2y - 5z = 15$	(5, 0, 0)	(0, $\frac{-15}{2}$, 0)	(0, 0, −3)
(d) $x + y + z = 0$ multiple answers	(1, 1, −2)	(2, 0, −2)	(0, 0, 0)

22. Five of the eight vertices of a cube are points:
A(-1,3,-2), B(4,3,-2), C(4,-2,-2), D(-1,-2,-2), and E(4,3,3). Find coordinates for the other three vertices.
(4, −2, 3); (−1, −2, 3); (−1, 3, 3)



23. Five of the vertices of a rectangular solid are points:
A(-1,-1,-5), B(-1,-1,2), C(-1,3,2), D(-1,3,-5), and E(1,-1,-5). Find the coordinates of the other three vertices.
(1, −1, 2); (1, 3, 2); (1, 3, −5)



Geometry
Worksheet 2 - Coordinates in Space

Name KEY
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Determine the distance between each pair of points, and determine the coordinates of the midpoint of the segment connecting them.

1. C(4, -8, 12) and D(7, 20, 18)

$d = \sqrt{(7-4)^2 + (20+8)^2 + (18-12)^2}$

$d = \sqrt{829}$

midpt $\left(\frac{4+7}{2}, \frac{-8+20}{2}, \frac{12+18}{2}\right)$

$= (5.5, 6, 15)$

2. E(3, 7, -1) and F(5, 7, 2)

$d = \sqrt{(5-3)^2 + (7-7)^2 + (2+1)^2}$

$d = \sqrt{13}$

midpt $\left(\frac{3+5}{2}, \frac{7+7}{2}, \frac{-1+2}{2}\right)$

$= (4, 7, \frac{1}{2})$

3. G(2, 2, 2) and H(-25, 4, 18)

$d = \sqrt{(2+25)^2 + (2-4)^2 + (2-18)^2}$

$d = \sqrt{989}$

Identify each of the following as true or false. If the statement is false, explain why.

4. Every point on the yz-plane has coordinates (c, y, z) for any real number c.

no, c must = 0

5. The point at (1, 8, -12) is inside the sphere $(x-3)^2 + (y-5)^2 + (z+2)^2 = 9$.

outside

center (3, 5, -2) $r = 3$

$d^2 = (3-1)^2 + (5-8)^2 + (-2+12)^2$

$d^2 = 113 \rightarrow d = 10.6 \rightarrow$ greater than $r = 3$

6. The intersection of the xy-plane, the yz-plane, and the xz-plane is the point (0, 0, 0).

True

7. The set of points in space 5 units from the point at (1, -1, 3) can be described by the equation: $(x-1)^2 + (y+1)^2 + (z-3)^2 = 25$.

True

8. The set of points equidistant from A(2, 5, 8) and B(-3, 4, 7) is a line that is the perpendicular bisector of $\overline{AB}$.

false, it is a plane

Determine the coordinates of the center and the measure of the radius for each sphere whose equation is given.

9. $x^2 + (y-3)^2 + (z+8)^2 = 81$

C (0, 3, -8) $r = 9$

10. $(x-5)^2 + (y+4)^2 + (z-10)^2 = 9$

C (5, -4, 10) $r = 3$

11. $x^2 + y^2 + (z-3)^2 = 49$

C (0, 0, 3) $r = 7$

12. $(x+4)^2 + (y-2)^2 + (z+12)^2 = 18$

C (-4, 2, -12) $r = \sqrt{18} = 3\sqrt{2}$

Write the equation of the sphere using the given information.

13. The center is at (-5, 11, -3), and the radius is 4.

$(x+5)^2 + (y-11)^2 + (z+3)^2 = 16$

14. The center is at (-2, 3, -4) and it contains the point at (5, -1, -1).

$r = \sqrt{(5+2)^2 + (-1-3)^2 + (-1+4)^2}$

$= \sqrt{74}$

$(x+2)^2 + (y-3)^2 + (z+4)^2 = 74$

15. The diameter has endpoints at (14, -8, 32) and (-12, 10, 12).

midpt $\left(\frac{14-12}{2}, \frac{-8+10}{2}, \frac{32+12}{2}\right)$

$= (1, 1, 22)$

$r = \sqrt{(-12-1)^2 + (10-1)^2 + (12-22)^2}$

$r = \sqrt{350}$

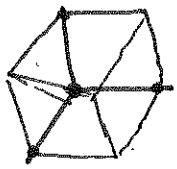
$(x-1)^2 + (y-1)^2 + (y-22)^2 = 350$

16. It is concentric with the sphere with equation $(x+5)^2 + (y-4)^2 + (z-19)^2 = 9$, and it has a radius of 6 units.

C (-5, 4, 19)

$(x+5)^2 + (y-4)^2 + (z-19)^2 = 36$

17. It is inscribed in a cube determined by the points at (0, 0, 0), (4, 0, 0), (0, 4, 0), and (4, 4, 4).



$r = 2$, C (2, 2, 2)

$(x-2)^2 + (y-2)^2 + (z-2)^2 = 4$

18. Find the perimeter of a triangle with vertices A(-1, 3, 2), B(0, 2, 4), and C(-2, 0, 3).

$AB = \sqrt{(0+1)^2+(2-3)^2+(4-2)^2} = \sqrt{6}$

$BC = \sqrt{(-2-0)^2+(0-2)^2+(3-4)^2} = 3$

$AC = \sqrt{(-2+1)^2+(0-3)^2+(3-2)^2} = \sqrt{11}$

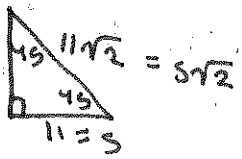
Perimeter $= \sqrt{6} + \sqrt{11} + 3$

19. Show that ΔABC is an isosceles right triangle if the vertices are A(3, 2, -3), B(5, 8, 6), and C(-3, -5, 3).

$AB = \sqrt{(5-3)^2+(8-2)^2+(6+3)^2} = 11$

$BC = \sqrt{(-3-5)^2+(-5-8)^2+(3-6)^2} = \sqrt{242} = \sqrt{2 \cdot 121} = 11\sqrt{2}$

$AC = \sqrt{(-3-3)^2+(-5-2)^2+(3+3)^2} = 11$



$s = 11 \cdot \frac{11\sqrt{2}}{11} = s\sqrt{2}$

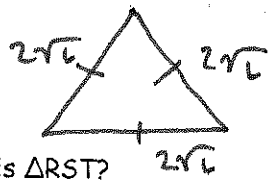
20. Consider R(6, 1, 3), S(4, 5, 5), and T(2, 3, 1).

(a) Determine the measures of $\overline{RS}$, $\overline{ST}$, and $\overline{RT}$.

$RS = \sqrt{(4-6)^2+(5-1)^2+(5-3)^2} = \sqrt{24} = 2\sqrt{6}$

$ST = \sqrt{(2-4)^2+(3-5)^2+(1-5)^2} = \sqrt{24} = 2\sqrt{6}$

$RT = \sqrt{(2-6)^2+(3-1)^2+(1-3)^2} = \sqrt{24} = 2\sqrt{6}$



(b) If $\overline{RS}$, $\overline{ST}$, and $\overline{RT}$ are sides of a triangle, what type of triangle is ΔRST?

equilateral

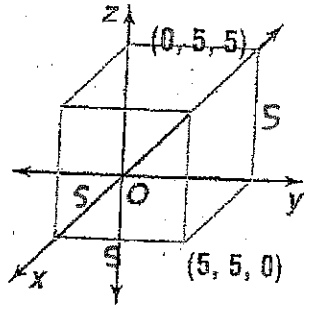
21. Find the surface area and volume of the rectangular prism at the right.

$TSA = 6(5 \cdot 5) = 150 \text{ units}^2$

$= 6s^2$

$Volume = l \cdot w \cdot h$

$= 5 \cdot 5 \cdot 5 = 125 \text{ units}^3$



22. Find z if the distance between R(5, 4, -1) and S(3, -2, z) is 7.

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$d^2 = (x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2$

$7^2 = (5-3)^2+(4+2)^2+(-1-z)^2$

$49 = 40 + 1 + 2z + z^2$

$z^2 + 2z - 8 = 0$

$(z+4)(z-2) = 0$

$z = -4, 2$

Answers:

1. CD= $\sqrt{829}$, midpt=(5.5, 6, 15)
2. EF= $\sqrt{13}$, midpt=(4, 7, 0.5)
3. GH= $\sqrt{989}$, midpt=(-11.5, 3, 10)
4. false; c must be zero
5. false; the point is outside the sphere
6. true
7. true
8. false; it is a plane containing the $\perp$ bisector of $\overline{AB}$
9. center(0, 3, -8); r=9
10. center(5, -4, 10); r=3
11. center(0, 0, 3); r=7
12. center(-4, 2, -12); r= $3\sqrt{2}$
13. $(x+5)^2+(y-11)^2+(z+3)^2 = 16$
14. $(x+2)^2+(y-3)^2+(z+4)^2 = 74$
15. $(x-1)^2+(y-1)^2+(z-22)^2 = 350$
16. $(x+5)^2+(y-4)^2+(z-19)^2 = 36$
17. $(x-2)^2+(y-2)^2+(z-2)^2 = 4$
18. perimeter= $\sqrt{6}+\sqrt{11}+3$
19. AB = 11 and AC = 11 so Δ is isosceles. $AB^2 + AC^2 = BC^2$ so Δ is a right Δ.
20. (a) RS = $\sqrt{24}$, ST= $\sqrt{24}$, RT= $\sqrt{24}$
    (b) it is equilateral
21. SA = 150 u^2; V = 125 u^3
22. z = 2 or z = -4