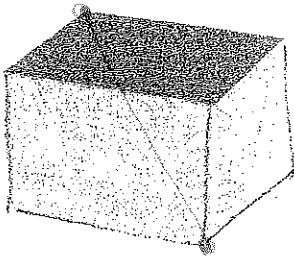


## Pythagorean Theorem in Three Dimensions

1. Find the length of the diagonal of a rectangular prism if  $l:w:h = 2:4:5$  and  $h = 15$ .

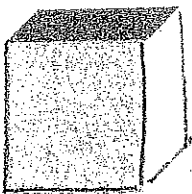


$$\begin{aligned}
 l^2 + w^2 + h^2 &= d^2 \\
 (2x)^2 + (4x)^2 + (5x)^2 &= d^2 \\
 6^2 + 12^2 + 15^2 &= d^2 \\
 36 + 144 + 225 &= d^2 \\
 405 &= d^2 \\
 9\sqrt{5} &= d
 \end{aligned}$$

Bottom front to opposite to corner

$$\begin{aligned}
 5x &= 15 \\
 x &= 3
 \end{aligned}$$

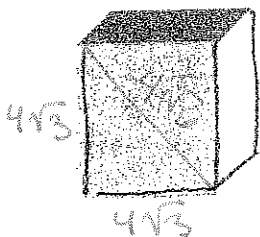
2. Find the height of a rectangular prism if its width is 5 in., its length is 7 in., and its diagonal is 12 in.



$$\begin{aligned}
 7^2 + 5^2 + h^2 &= 12^2 \\
 49 + 25 + h^2 &= 144 \\
 h^2 &= 70 \\
 h &= \sqrt{70}
 \end{aligned}$$

$$\begin{array}{r}
 70 \\
 \swarrow \searrow \\
 2 \quad 35 \\
 \swarrow \searrow \\
 7 \quad 5
 \end{array}$$

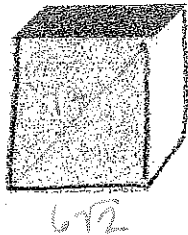
3. The length of the diagonal of a cube is 12. Find the length of a diagonal of a face.



$$\begin{aligned}
 3(x^2) &= 12^2 \\
 3x^2 &= 144 \\
 x^2 &= 48 \\
 x &= 4\sqrt{3}
 \end{aligned}$$

$$\begin{array}{r}
 48 \\
 \swarrow \searrow \\
 12 \quad 4 \\
 \swarrow \searrow \\
 4 \quad 3 \quad 2 \quad 2 \\
 \swarrow \searrow \\
 2 \quad 2
 \end{array}$$

4. The length of the diagonal of a face of a cube is 12. Find the length of the diagonal of the cube.



$$s\sqrt{2} = 12$$

$$s = \frac{12}{\sqrt{2}}$$

$$s = 6\sqrt{2}$$

$$(6\sqrt{2})^2 + (6\sqrt{2})^2 + (6\sqrt{2})^2 = d^2$$

$$3(6\sqrt{2})^2 = d^2$$

$$3(36 \cdot 2) = d^2$$

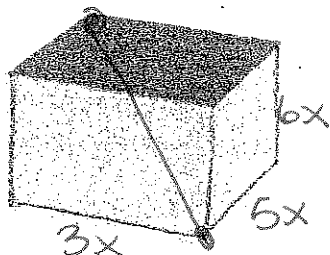
$$3 \cdot 72 = d^2$$

$$216 = d^2$$

$$6\sqrt{6} = d$$

216  
11

5. The length, width, and height of a rectangular prism are in the ratio of 3:5:6. If the diagonal is 20 in. long, find the three dimensions.



$$(3x)^2 + (5x)^2 + (6x)^2 = 20^2$$

$$9x^2 + 25x^2 + 36x^2 = 400$$

$$70x^2 = 400$$

$$x^2 = \frac{400}{70}$$

$$x = \sqrt{\frac{400}{70}}$$

$$x = \frac{20}{\sqrt{70}}$$

$$x = \frac{20\sqrt{70}}{70}$$

$$x = \frac{2\sqrt{70}}{7}$$

70  
11  
2 35  
11  
7 5

$$L = \frac{6\sqrt{70}}{7}$$

$$W = \frac{10\sqrt{70}}{7}$$

$$h = \frac{12\sqrt{70}}{7}$$